

State the amplitude and period for each function.

1. $y = 3\sin 2x$

amp = 3

Period = $\frac{2\pi}{2} = \pi$

Describe the transformation for each function.

7. $y = \sin(x - \pi)$

P.S. π or $\pi + \pi$

2. $y = \frac{5}{2} \cos(\frac{x}{2})$

amp = $\frac{5}{2}$

Per = $\frac{2\pi}{\frac{1}{2}} = 4\pi$

8. $y = \cos x + 1$

V.S. 1 or up 1

3. $y = \frac{1}{2} \sin(\frac{\pi x}{3})$

amp = $\frac{1}{2}$

Per = $\frac{2\pi}{\frac{\pi}{3}} = 6$

9. $y = \sin(x - \frac{\pi}{2}) - 3$

P.S. $-\frac{\pi}{2}$ or $\pi + \frac{\pi}{2}$

V.S. -3 or down 3

Write the equation of the function with the given information.

12. Sine function

V.S.: 3

P.S.: π

13. Cosine function

V.S.: 2

P.S.: 4π

14. Sine function

V.S.: 2

P.S.: π

15. Cosine function

V.S.: 3

P.S.: $\frac{\pi}{2}$

$f(x) = \sin(x - \pi) + 3$

$f(x) = \cos(x - 4\pi) + 2$

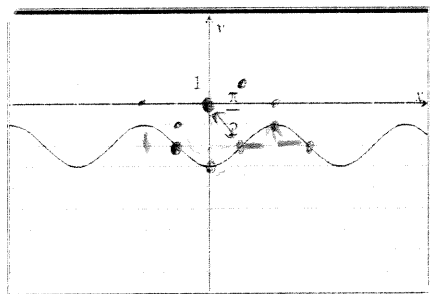
$f(x) = \sin(x - \pi) + 2$

$f(x) = \cos(x - \frac{\pi}{2}) + 3$

16. The parent function is sine.

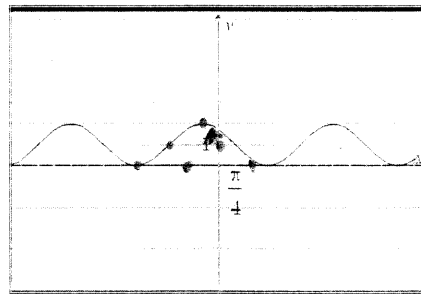
1 possible answer

$f(x) = \sin(x - \frac{\pi}{2}) - 2$



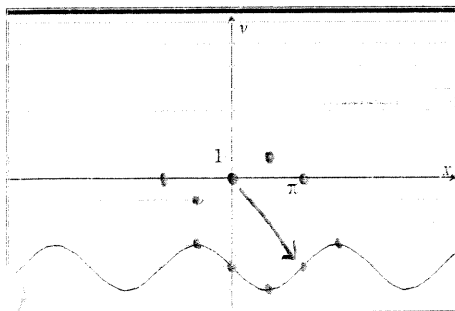
17. The parent function is cosine.

$f(x) = \cos(x + \frac{\pi}{4}) + 1$



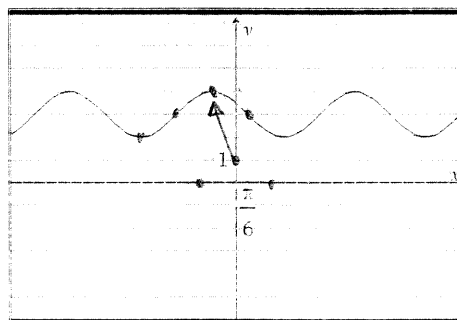
18. The parent function is sine.

$f(x) = \sin(x - \pi) - 4$



19. The parent function is cosine.

$f(x) = \cos(x + \frac{\pi}{3}) + 1$



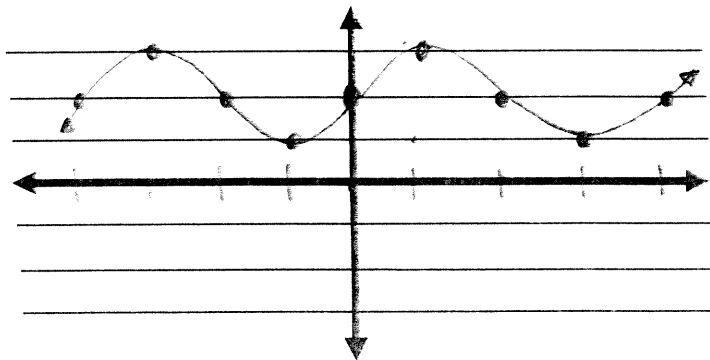
Graph the following. Label the axes. Provide the period, phase shift, and vertical shift in the space provided. If any of these do not exist, write *none*. Show at least one positive and negative period.

20. $y = \sin x + 2$

Period = 2π

P. S. = *none*

V. S. = 2



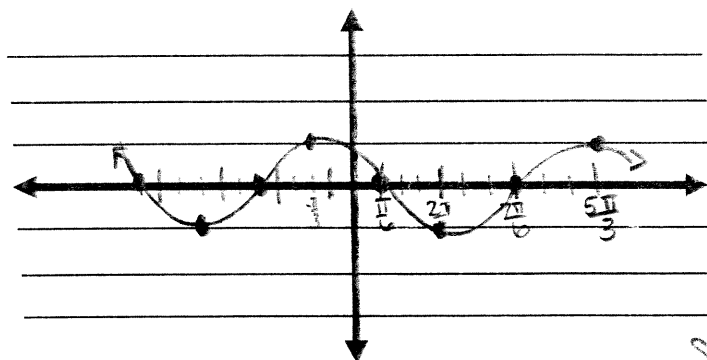
21. $y = \cos(x + \frac{\pi}{3})$

Period = 2π

P. S. = $-\frac{\pi}{3}$

V. S. = *none*

| x | y |
|-------------------|----|
| $-\frac{2\pi}{3}$ | 1 |
| $-\frac{\pi}{3}$ | 0 |
| 0 | -1 |
| $\frac{\pi}{3}$ | 0 |
| $\frac{2\pi}{3}$ | 1 |



Tricky one
Since you have a variation in denom
Choose the smallest one and
Skip tic marks when needed.

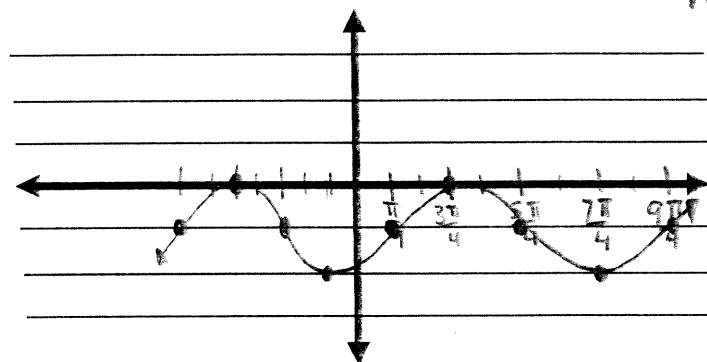
22. $y = \sin(x - \frac{\pi}{4}) - 1$

Period = 2π

P. S. = $\frac{\pi}{4}$

V. S. = -1

| x | y |
|------------------|----|
| $\frac{\pi}{4}$ | -1 |
| $\frac{3\pi}{4}$ | 0 |
| $\frac{5\pi}{4}$ | -1 |
| $\frac{7\pi}{4}$ | 0 |
| $\frac{9\pi}{4}$ | -1 |



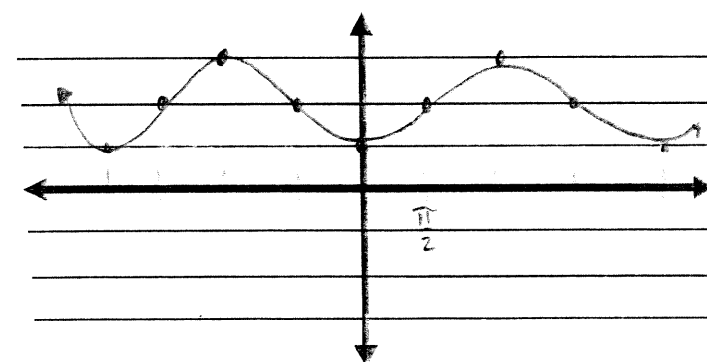
23. $y = \cos(x + \pi) + 2$

Period = 2π

P. S. = $-\pi$

V. S. = 2

| x | y |
|------------------|---|
| $-\pi$ | 3 |
| $-\frac{\pi}{2}$ | 2 |
| 0 | 1 |
| $\frac{\pi}{2}$ | 2 |
| π | 3 |



Danielle

State the amplitude and period for each function.

1. $y = 3\sin 2x$

A: 3

P: π

2. $y = \frac{5}{2}\cos(\frac{x}{2})$

A: $5/2$

P: 4π

3. $y = \frac{1}{2}\sin(\frac{\pi x}{3})$

A: $1/2$

P: 6

4. $y = -2\sin x$

A: -2

P: 2π

5. $y = 3\sin 10x$

A: 3

P: $\frac{\pi}{5}$

6. $y = \frac{1}{2}\cos \frac{2x}{3}$

A: $1/2$

P: 3π

Describe the transformation for each function.

7. $y = \sin(x-\pi)$

8. $y = -\cos 2x$

9. $y = \sin 2x + 3$

10. $y = 2\cos(x + \frac{\pi}{2})$

11. $y = 2\cos(2\pi x - 4)$

Write the equation of the function with the given information.

12. Sine function

Amp: 3

Period: π

13. Cosine function

Amp: 2

Period: 4π

14. Sine function

Amp: 2

Period: π

Phase shift: 2

15. Cosine function

Amp: 3

Period: $\frac{\pi}{2}$

Phase shift: -2

Graph the following. Label the axes. Provide the amplitude, period, phase shift, and vertical shift in the space provided. If any of these do not exist, write *none*. Show at least one positive and negative period.

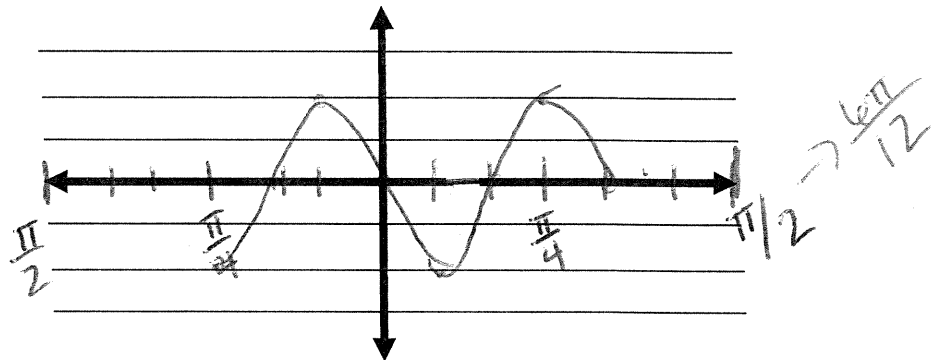
16. $y = -2\sin 6x$

Amp. = -2

Period = $\pi/6$ $\frac{\pi}{3}$

P. S. = —

V. S. = —



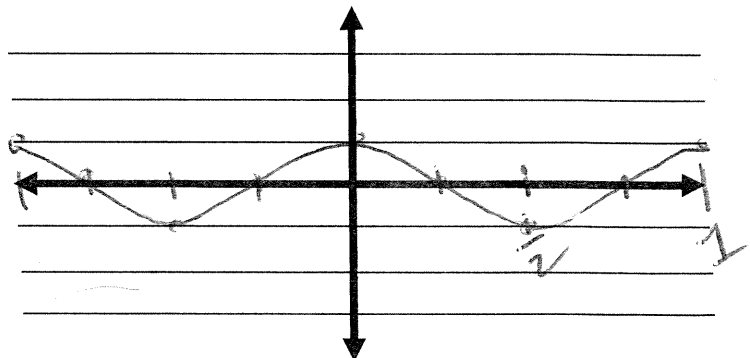
17. $y = \cos 2\pi x$

Amp. = 1

Period = 1

P. S. = —

V. S. = —



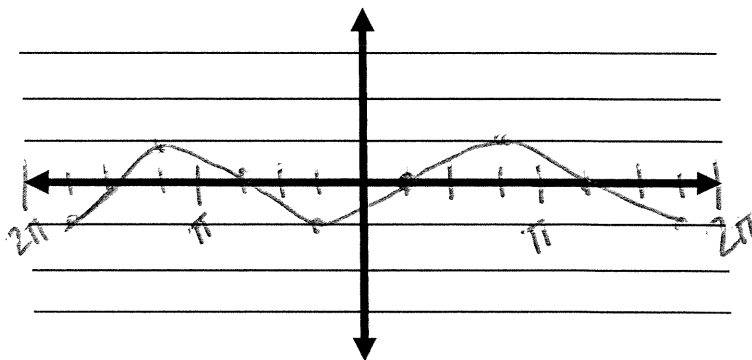
18. $y = \sin(x - \frac{\pi}{4})$

Amp. = 1

Period = 2π

P. S. = $+\pi/4$

V. S. =



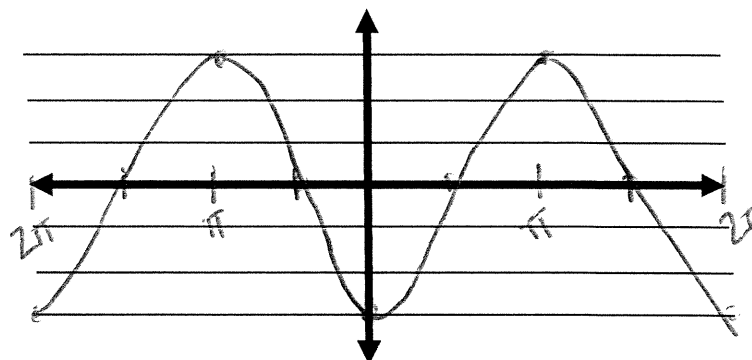
19. $y = 3\cos(x + \pi)$

Amp. = 3

Period = 2π

P. S. = $-\pi$

V. S. =



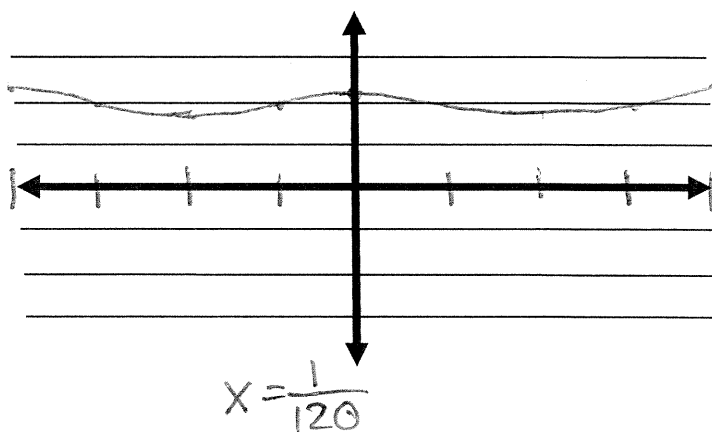
20. $y = 2 + \frac{1}{10}\cos 60\pi x$

Amp. = $1/10$

Period = $1/30$

P. S. =

V. S. = $+2$



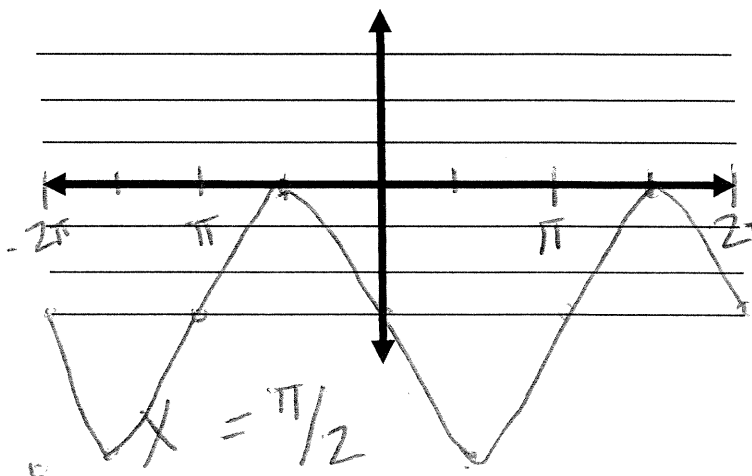
21. $y = 3\sin(x + \pi) - 3$

Amp. = 3

Period = 2π

P. S. = $-\pi$

V. S. = -3



Find two solutions of the equation. Give answers in degrees ($0^\circ \leq \theta < 360^\circ$) and radians ($0 \leq \theta < 2\pi$).

1. $\sin\theta = \frac{1}{2}$ 2. $\cos\theta = \frac{\sqrt{2}}{2}$ 3. $\csc\theta = -\sqrt{2}$ 4. $\cot\theta = -\sqrt{3}$ 5. $\sec\theta = \sqrt{2}$

Find the exact value of each expression. Do not use a calculator.

6. $\tan \frac{3\pi}{2}$ 7. $\cos \frac{5\pi}{3}$ 8. $\sin \frac{3\pi}{4}$ 9. $\sec \frac{\pi}{6}$ 10. $\csc \frac{7\pi}{6}$

11. $\cos\theta = -\frac{2}{3}$ and $\tan\theta > 0$, $\sin\theta =$ _____ and $\tan\theta =$ _____

12. $\sec\theta = \frac{8}{3}$ and $\csc\theta < 0$, $\tan\theta =$ _____ and $\sin\theta =$ _____

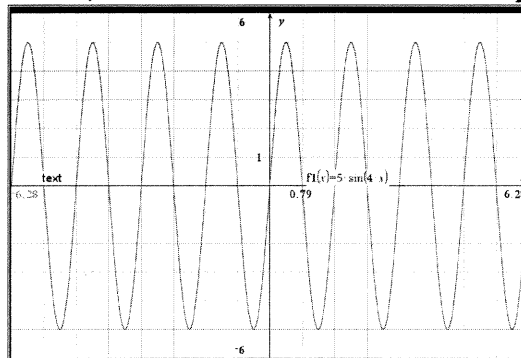
Graph the following. Label the axes. Provide the amplitude, period, phase shift, and vertical shift in the space provided. If any of these do not exist, write *none*. Show at least one positive and negative period.

13. $y = 5 \sin 4x$
Amp. = 5

Period = $\pi/2$

P. S. = —

V. S. = —



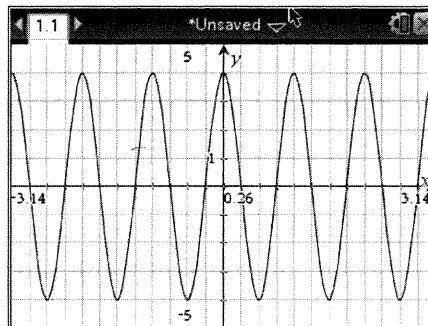
14. $y = 4 \cos 6x$

Amp. = 4

Period = $\pi/3$

P. S. = —

V. S. = —



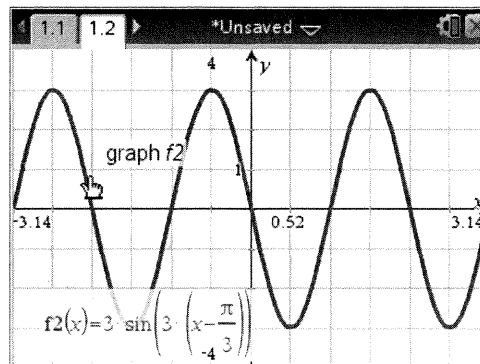
15. $y = 3 \sin(3x - \pi)$

Amp. = 3

Period = $2\pi/3$

P. S. = $\pi/3$

V. S. = —



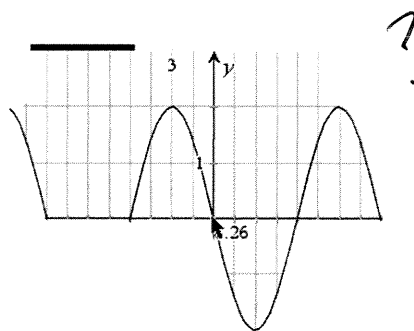
16. $y = 2\cos(3x + \frac{\pi}{2})$ $3(x + \frac{\pi}{6})$

Amp. = 2

Period = $\frac{2\pi}{3}$

P. S. = $-\frac{\pi}{6}$

V. S. =



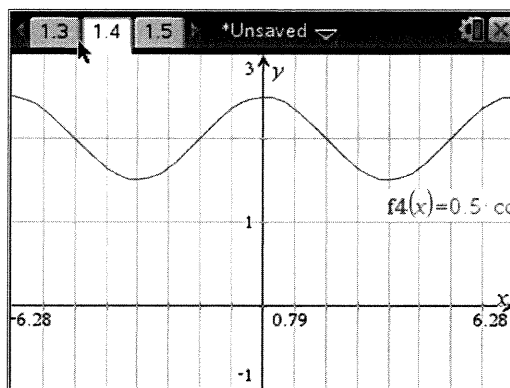
17. $y = \frac{1}{2} \cos x + 2$

Amp. = $\frac{1}{2}$

Period = 2π

P. S. =

V. S. = +2



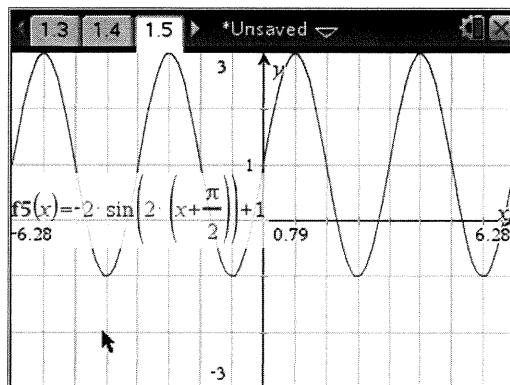
18. $y = -2\sin(2x + \frac{\pi}{2}) + 1$

Amp. = 2

Period = π

P. S. = $-\frac{\pi}{2}$

V. S. = +1



Write the equation of a sine function that has the given characteristics.

19. amp: 3
Period: π

$3\sin 2x$

20. Amp: 4
period: 1

$4\sin(2\pi x)$

21. Amp: 2
period: π
Phase shift: -2

$2\sin(2(x+2))$

Write the equation of a cosine function that has the given characteristics.

22. amp: 2
Period: $\frac{\pi}{6}$

$2\cos(x - \frac{\pi}{6})$

23. Amp: 3
phase shift: 4
Reflect over the x-axis

$-3\cos(x - 4)$

24. Amp: $\frac{1}{2}$
period: 6π
vertical shift: up 4

$\frac{1}{2}(\cos(\frac{1}{3}(x))) + 4$

10.

| x | y | x | y |
|------------------|----|------------------|----|
| 0 | 0 | $-\frac{\pi}{2}$ | 0 |
| $\frac{\pi}{2}$ | 1 | | 2 |
| π | 0 | π | 0 |
| $\frac{3\pi}{2}$ | -1 | $\frac{5\pi}{2}$ | -2 |
| 2π | 0 | $\frac{7\pi}{2}$ | 0 |
| 3π | 0 | | |

$$6\pi = \frac{12\pi}{2}$$

$$3\pi = \frac{6\pi}{2}$$

11.

| x | y | tan |
|------------------|-------|-----|
| 0 | 0 | |
| $\frac{\pi}{4}$ | 1 | |
| $\frac{\pi}{2}$ | undef | |
| $\frac{3\pi}{4}$ | -1 | |
| π | 0 | |

| x | y | cot |
|------------------|-------|-----|
| 0 | undef | |
| $\frac{\pi}{4}$ | 1 | |
| $\frac{\pi}{2}$ | 0 | |
| $\frac{3\pi}{4}$ | -1 | |
| π | undef | |

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$$\frac{\pi_1}{\pi_2} = \frac{1}{2}$$

$$\frac{\pi_1}{\pi_2} = \frac{1}{2}$$

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| 99 | 1 | 1 |

16.

| X | Y |
|------------------|----|
| 0 | 0 |
| $\frac{\pi}{2}$ | 1 |
| π | 0 |
| $\frac{3\pi}{2}$ | -1 |
| 2π | 0 |
| $\frac{1}{6}$ | -2 |

16.

| X | Y |
|-------------------|----|
| 0 | 0 |
| $\frac{\pi}{12}$ | -2 |
| $\frac{2\pi}{12}$ | 0 |
| $\frac{3\pi}{12}$ | 2 |
| $\frac{4\pi}{12}$ | 0 |

$$a \cdot \cos(bx + c) + d$$

$$a \cdot \cos\left(b\left(x - \frac{c}{b}\right) + d\right)$$

17.

| X | Y |
|------------------|----|
| 0 | 1 |
| $\frac{\pi}{2}$ | 0 |
| π | -1 |
| $\frac{3\pi}{2}$ | 0 |
| 2π | 1 |
| $\frac{1}{2\pi}$ | |

| X | Y |
|---------------|----|
| 0 | 1 |
| $\frac{1}{4}$ | 0 |
| $\frac{1}{2}$ | -1 |
| $\frac{3}{4}$ | 0 |
| 1 | 1 |

18.

| x | y |
|-----------------|---|
| $\frac{\pi}{4}$ | |

✓
don't
be lazy

19.

| x | y |
|----------|----|
| 0 | 1 |
| $\pi/2$ | 0 |
| π | -1 |
| $3\pi/2$ | 0 |
| 2π | 1 |
| $-\pi$ | 0 |

| x | y |
|----------|----|
| $-\pi$ | 3 |
| $-\pi/2$ | 0 |
| 0 | -3 |
| $\pi/2$ | 0 |
| π | 3 |

20.

| x | y |
|----------|----|
| 0 | 1 |
| $\pi/2$ | 0 |
| π | -1 |
| $3\pi/2$ | 0 |
| 2π | 1 |

 $\frac{1}{60\pi}$
 $\frac{1}{10} + 2$
 $\frac{2}{120}$
 $\frac{3}{120}$
 $\frac{4}{120}$

x | y

| | |
|-----------------|-----|
| 0 | 2.1 |
| $\frac{1}{120}$ | 2 |
| $\frac{1}{60}$ | 1.9 |
| $\frac{1}{40}$ | 2 |
| $\frac{1}{30}$ | 2.1 |

21.

| x | y |
|----------|----|
| 0 | 0 |
| $\pi/2$ | 1 |
| π | 0 |
| $3\pi/2$ | -1 |
| 2π | 0 |
| $-\pi$ | 0 |

| x | y |
|----------|----|
| $-\pi$ | -3 |
| $-\pi/2$ | 0 |
| 0 | -3 |
| $\pi/2$ | -6 |
| π | -3 |

4.6A Worksheet

NO CALCULATOR

Find the exact values for each of the following:

1. $\cos \frac{\pi}{4}$ _____ 2. $\sin \frac{3\pi}{2}$ _____ 3. $\tan \frac{5\pi}{4}$ _____

4. $\sin \frac{2\pi}{3}$ _____ 5. $\tan \frac{7\pi}{6}$ _____ 6. $\cos \frac{5\pi}{3}$ _____

For each of the following, solve for θ ($0 \leq \theta < 2\pi$)

7. $\sin \theta = \frac{\sqrt{3}}{2}$ _____

8. $\cos \theta = -\frac{\sqrt{3}}{2}$ _____

9. $\tan \theta = -1$ _____

Graph the following. Label the axes. Provide the amplitude, period, phase shift, and vertical shift in the space provided. If any of these do not exist, write *none*. Show at least one complete period of each graph.

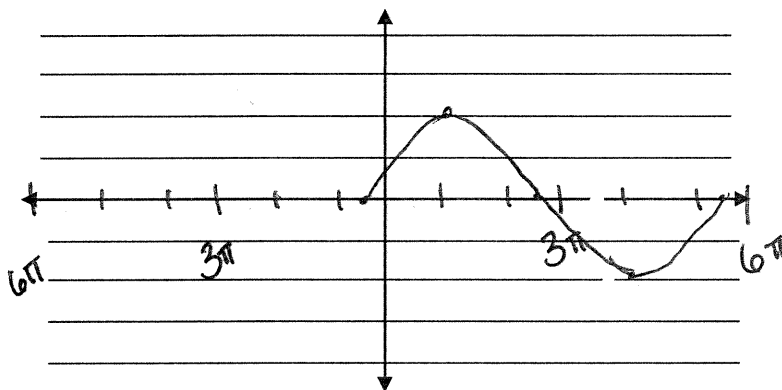
10. $y = 2 \sin \frac{1}{3} \left(x + \frac{\pi}{2} \right)$

Amp. = 2

Period = 6π

H. S. = $-\pi/2$

V. S. = —

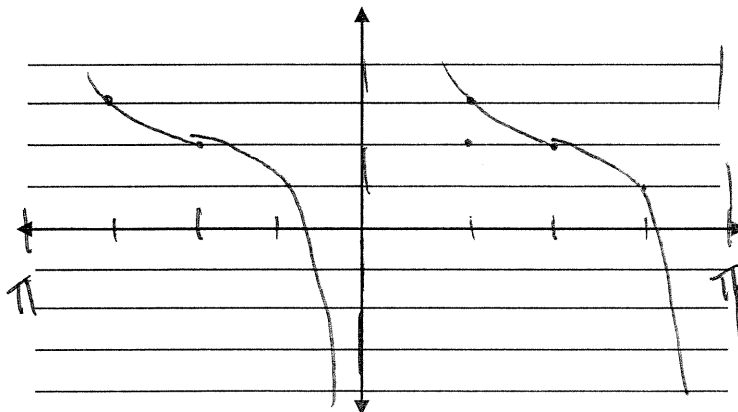


11. $f(x) = \cot(x) + 2$

Period = π

H. S. = —

V. S. = +2

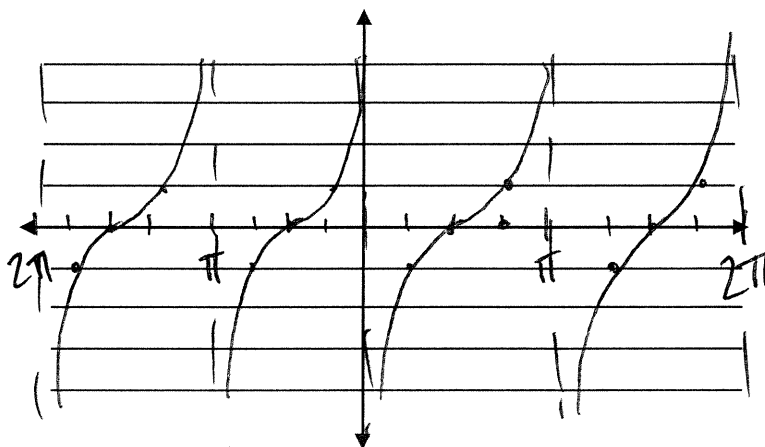


12. $y = \tan\left(x - \frac{\pi}{2}\right)$

Period = π

H. S. = π

V. S. = $+\pi/2$

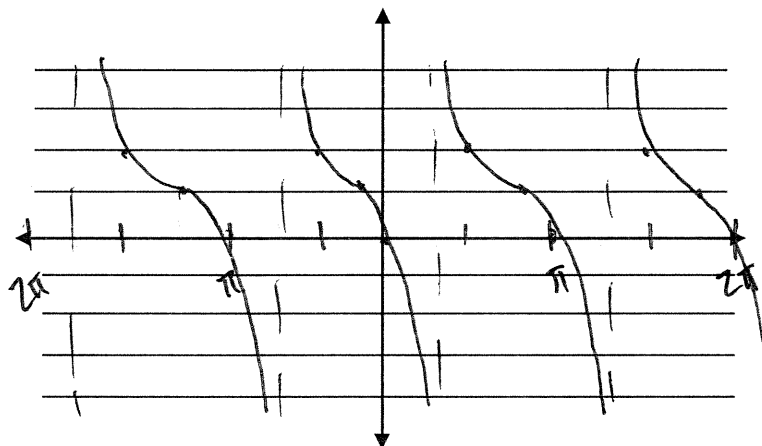


13. $f(x) = \cot(x - \pi) - 1$

Period = π

H. S. = $+\pi$

V. S. = -1

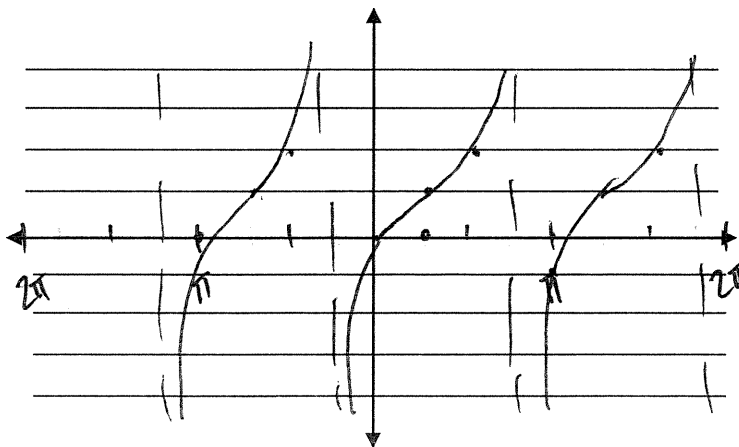


14. $f(x) = \tan\left(x - \frac{\pi}{4}\right) + 1$

Period = π

H. S. = $+\pi/4$

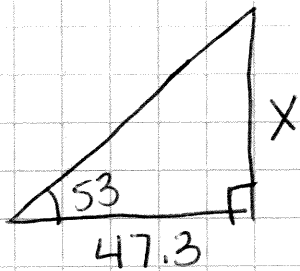
V. S. = $+1$



RIGHT TRIANGLE WORKSHEET

1. Suppose you have been assigned to measure the height of the local water tower. Climbing makes you dizzy, so you decide to do the whole job at ground level. From a point 47.3 meters from the base of the water tower, you find that you must look up at an angle of 53° to see the top of the tower. How tall is the tower? Draw the triangle.
2. A ship is passing through the Strait of Gibraltar. At its closest point of approach, Gibraltar radar determines that it is 2400 meters away. Later, the radar determines that it is 2650 meters away. By what angle did the ship's bearing from Gibraltar change? How far did the ship travel during the two observations?
3. You lean a ladder 6.7 meters long against the wall. It makes an angle of 63° with the level ground. How high up is the top of the ladder?
4. You must order a new rope for the flagpole. To find out what length of rope is needed, you observe that pole casts a shadow 11.6 meters long on the ground. The angle between the sun's rays and the ground is 36.8° . How tall is the pole?
5. Your cat is trapped on a tree branch 6.5 meters above the ground. Your ladder is only 6.7 meters long. If you place the ladder's tip on the branch, what angle will the ladder make with the ground?
6. The tallest freestanding structure in the world is the 553 meter tall CN tower in Toronto, Ontario. Suppose that at a certain time of day it casts a shadow 1100 meters long on the ground. What is the angle of elevation of the sun at that time of day?
7. Scientists estimate the heights of features on the moon by measuring the lengths of the shadows they cast on the moon's surface. From a photograph, you find that the shadow cast on the inside of a crater by its rim is 325 meters long. At the time the photograph was taken, the sun's angle to the horizontal surface was 23.6° . How high does the rim rise above the inside of the crater?
8. A beam of gamma rays is to be used to treat a tumor known to be 5.7 cm beneath the patient's skin. To avoid damaging a vital organ, the radiologist moves the source over 8.3 cm. At what angle to the patient's skin must the radiologist aim the gamma ray source to hit the tumor? How far will the gamma rays have to pass through the body to hit the tumor?
9. When surveyors measure land that slopes significantly, the distance which is measured will be longer than the horizontal distance which must be drawn on a map. Suppose that the distance from the top edge of the Okapi Creek bed to the edge of the water is 37.8 meters. The land slopes downward at 27.6° to the horizontal. a) What is the horizontal distance from the top of the banks to the edge of the creek? b) How far is the surface of the creek below the level of the surrounding land?
10. From a point on the North Rim of the Grand Canyon, a surveyor, sighting the South Rim a little below the North Rim, measures the angle to be 1.3° . From an aerial photograph, he determines that the horizontal distance between the two points is 1600 meters. How many meters is the South Rim below the North Rim?
11. A submarine at the surface of the ocean makes an emergency dive, its path making an angle of 21° with the surface. If it goes for 300 meters along its downward path, how deep will it be? What horizontal distance is it from its starting point?
12. An observer 5.2 kilometers from the launch pad observes a missile ascending. At a particular, the angle of elevation is 37.6° . How high is the missile?
13. Suppose that you are on a salvage ship in the Gulf of Mexico. Your sonar system has located a sunken Spanish galleon at a slant distance of 683 meters from your ship, with an angle to the horizontal of 27.8° . How deep is the water at the galleon's location? How far must you sail to be directly above the galleon?

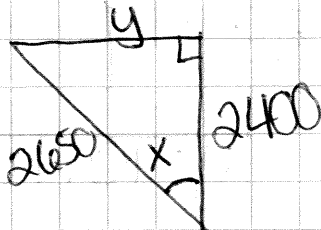
1.



$$\tan 53 = \frac{x}{47.3}$$

$$x = 62.77 \text{ m}$$

2.

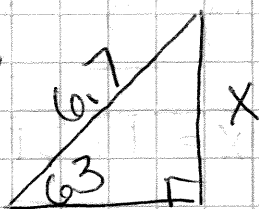


$$\cos(x) = \frac{2400}{2650}$$

$$x = 25.1^\circ$$

$$y^2 = 2650^2 - 2400^2 \Rightarrow \underline{1123.6 \text{ m}}$$

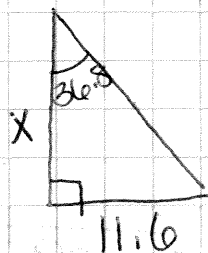
3.



$$\sin 63 = \frac{x}{6.7}$$

$$x = 5.97 \text{ m}$$

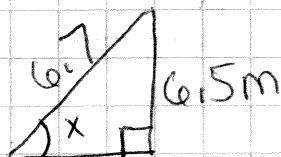
4.



$$\tan 36.8 = \frac{11.6}{x}$$

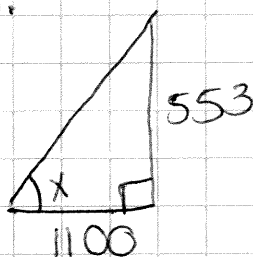
$$x = 15.5 \text{ m}$$

5.

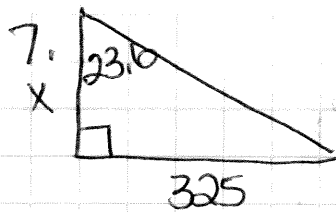


$$\sin^{-1}\left(\frac{6.5}{6.7}\right) = x = 76^\circ$$

6.

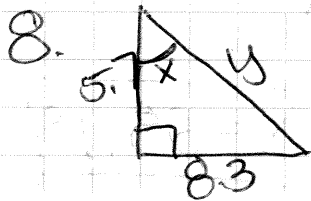


$$\tan^{-1}\left(\frac{553}{1100}\right) = \underline{26.7^\circ}$$



$$\tan 23.6 = \frac{325}{x}$$

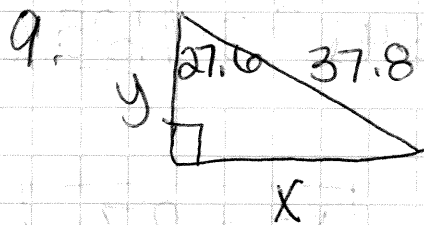
$$\boxed{x = 743.9 \text{ m}}$$



$$\tan^{-1}\left(\frac{8.3}{5.7}\right) = 55.5^\circ$$

$$\cos 55.5 = \frac{5.7}{y}$$

$$\boxed{y = 10.1 \text{ cm}}$$

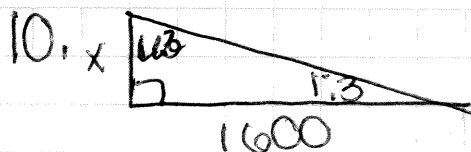


$$\cos 27.6 = \frac{y}{37.8}$$

$$\boxed{y = 33.5 \text{ m}}$$

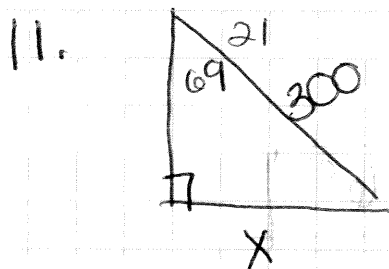
$$\sin 27.6 = \frac{x}{37.8}$$

$$\boxed{x = 17.5 \text{ m}}$$



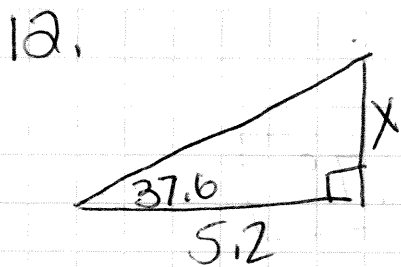
$$\tan 1.3 = \frac{1600}{x}$$

$$\boxed{x = 36.3 \text{ m}}$$



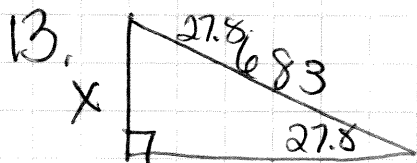
$$\sin 69 = \frac{x}{300}$$

$$\boxed{x = 280 \text{ m}}$$



$$\tan 37.6 = \frac{x}{5.2}$$

$$\boxed{x = 4 \text{ km}}$$



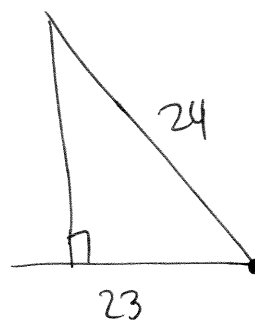
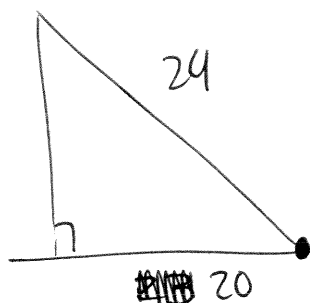
$$\sin 27 = \frac{x}{683}$$

$$\boxed{x = 310 \text{ m}}$$

Round all answers to the nearest hundredth.

1. A 24 foot ladder is leaning against a wall. The manufacturer states that the maximum horizontal reach for the ladder is 23 ft. If the ladder is leaning on a wall 20ft from the ground, is it following the specifications of the manufacturer? Why or why not. What is the angle of elevation for the maximum horizontal reach for the ladder?

16.60°
yes



2. A man on a 135-ft vertical cliff looks down at an angle of 16 degrees and sees his friend. How far away is the man from his friend? How far is the friend from the base of the cliff? Is this an accurate length? What other information would you need if it isn't accurate?

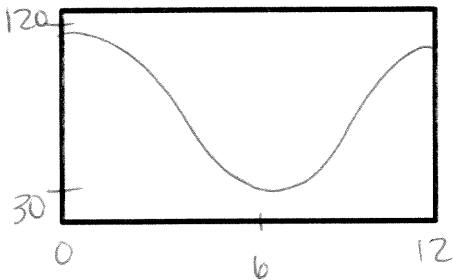
489.77 ft away from friend
470.8 ft away from base

3. Amelia sees a jet heading south away from her at 42 degree angle of elevation. Twenty seconds later the jet is still moving away from her, heading south at a 15 degree angle of elevation. If the jets elevation is constantly 6.3 km, how fast is it flying in kilometers per hour?

2972.7 km/hr

1. A company that produces snowboards, which are seasonal products, forecasts monthly sales for 1 year to be $S = 74.50 + 43.75 \cos \frac{\pi t}{6}$, where S is the sales in thousands of units and t is the time in months, with $t=1$ corresponding to January. In a one year period, what month did the maximum and minimum sales occur? How many units were sold in these months?

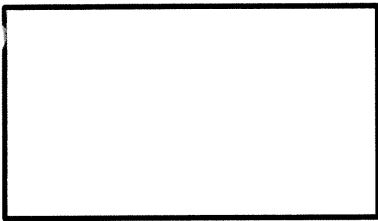
Sketch the graph.



Max - December (118 thousand)
Min - June (30.8 thousand)

2. You are riding a Ferris wheel. Your height h (in feet) above the ground at any time t (in seconds) can be modeled by $h = 25 \sin \frac{\pi}{15}(t - 75) + 30$. The Ferris wheel turns for 135 seconds before it stops to let the first passengers off. What are the maximum and minimum heights above the ground?

Sketch the graph.

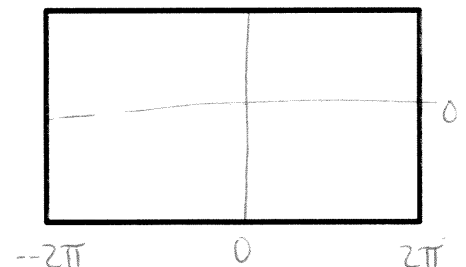
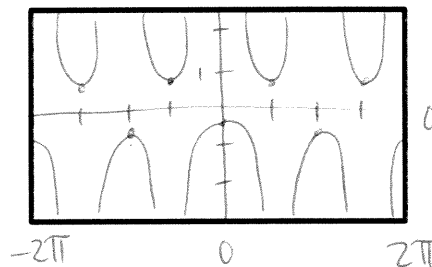
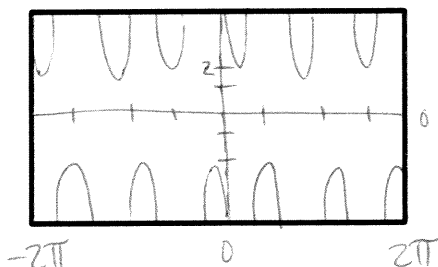


Graph the following on your graphing calculator. Sketch the graph on the interval of $[-2\pi, 2\pi]$.

3. $y = 2 \csc 3x$

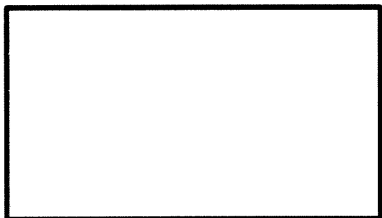
4. $y = \frac{1}{2} \sec(2x - \pi)$

5. $y = |x \sin x|$

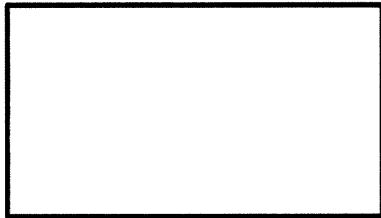


Use the graph of the function to approximate the solution(s) to the equation on the interval $[-2\pi, 2\pi]$.
Sketch your graph.

6. $\cot x = 1$



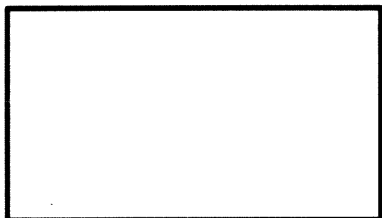
7. $\tan x = -\sqrt{3}$



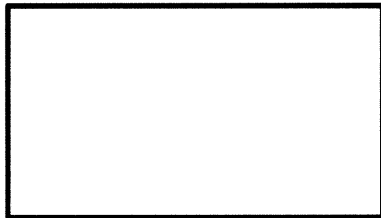
8. $\sec x = -2$



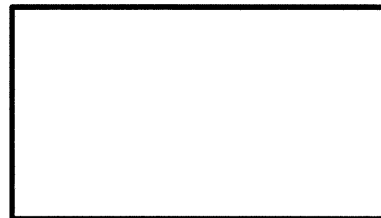
9. $\csc x = \sqrt{2}$



10. $x \cos x = 2$

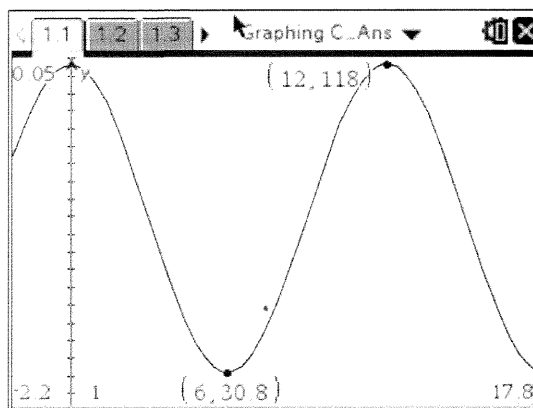


11. $-2 = |x| \sin x$



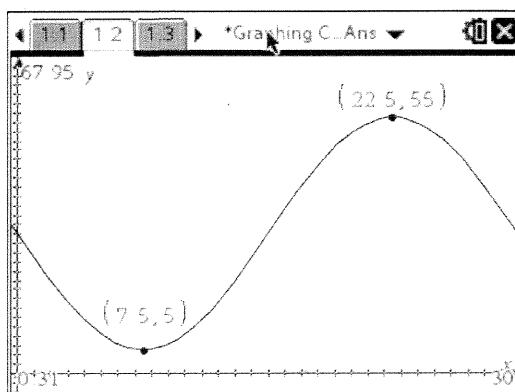
1. A company that produces snowboards, which are seasonal products, forecasts monthly sales for 1 year to be $S = 74.50 + 43.75 \cos \frac{\pi t}{6}$, where S is the sales in thousands of units and t is the time in months, with $t=1$ corresponding to January. In a one year period, what month did the maximum and minimum sales occur? How many units were sold in these months?

Sketch the graph.



2. You are riding a Ferris wheel. Your height h (in feet) above the ground at any time t (in seconds) can be modeled by $h = 25 \sin \frac{\pi}{15}(t - 75) + 30$. The Ferris wheel turns for 135 seconds before it stops to let the first passengers off. What are the maximum and minimum heights above the ground?

Sketch the graph.

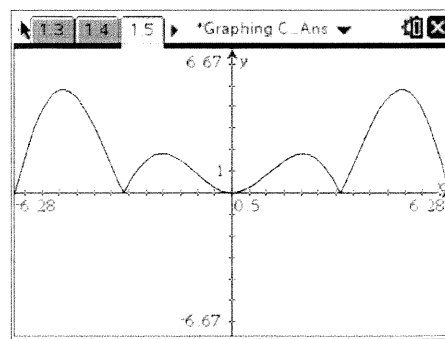
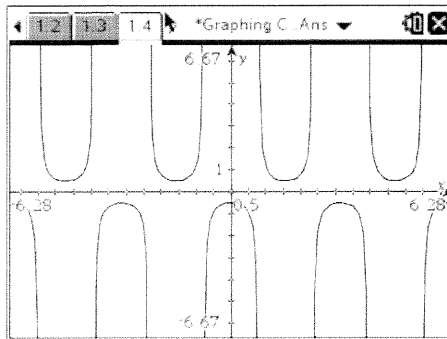
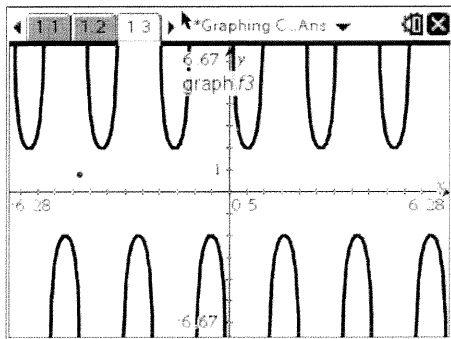


Graph the following on your graphing calculator. Sketch the graph on the interval of $[-2\pi, 2\pi]$.

3. $y = 2 \csc 3x$

4. $y = \frac{1}{2} \sec(2x - \pi)$

5. $y = |x \sin x|$



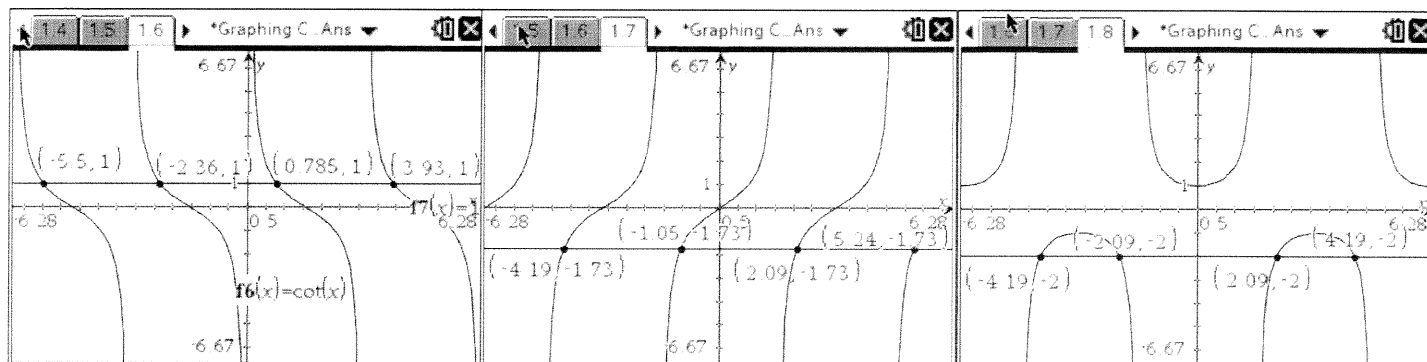
Use the graph of the function to approximate the solution(s) to the equation on the interval $[-2\pi, 2\pi]$.

Sketch your graph.

6. $\cot x = 1$

7. $\tan x = -\sqrt{3}$

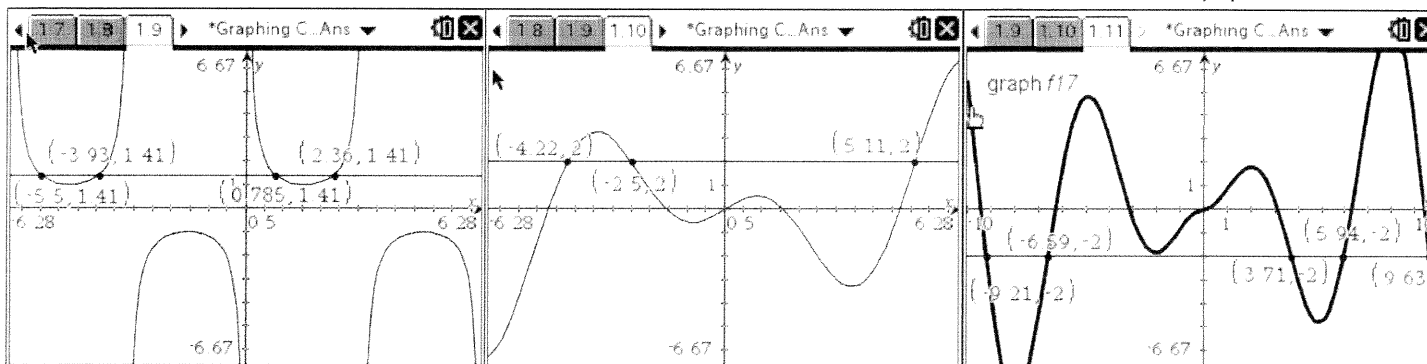
8. $\sec x = -2$



9. $\csc x = \sqrt{2}$

10. $x \cos x = 2$

11. $-2 = |x| \sin x$



Precalculus
Graphing Review
NO CALCULATOR

Name _____

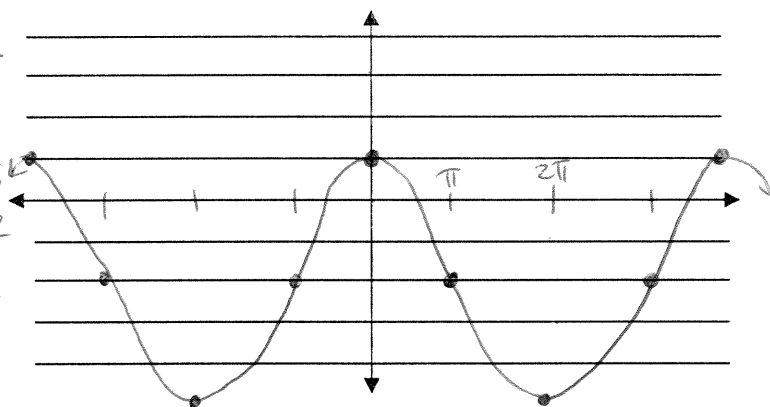
Graph the following. Label the axes. Provide the amplitude, period, phase shift, and vertical shift in the space provided. If any of these do not exist, write *none*. Show at least one complete period of each graph.

1. $y = 3 \cos\left(\frac{1}{2}x\right) - 2$

| X | Y |
|----------|--------|
| 0 | 0 |
| $\pi/2$ | π |
| π | 2π |
| $3\pi/2$ | 3π |
| 2π | 4π |

Amp. = 3
Period = 4π
H. S. = 0
V. S. = -2

| y | Y |
|----|----|
| 1 | 1 |
| 0 | -2 |
| -1 | -5 |
| 0 | -2 |
| 1 | 1 |

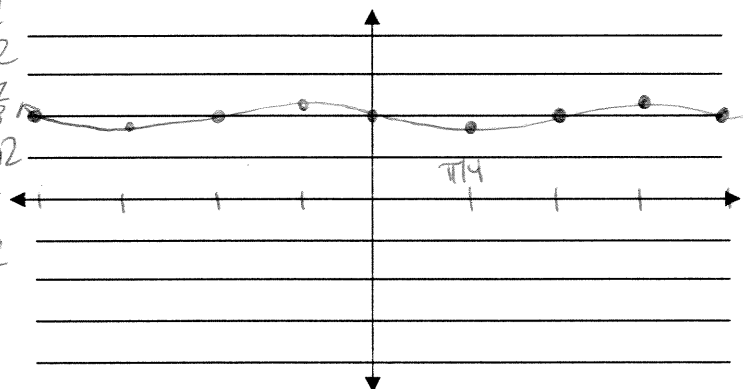


2. $y = -\frac{1}{3} \sin\left(2\left(x - \frac{\pi}{2}\right)\right) + 2$

| X | Y |
|----------|----------|
| 0 | $\pi/2$ |
| $\pi/2$ | $3\pi/4$ |
| π | π |
| $3\pi/2$ | $5\pi/4$ |
| 2π | $3\pi/2$ |

Amp. = $+\frac{1}{3}$
Period = π
H. S. = $\frac{\pi}{2}$
V. S. = 2

| y | Y |
|----|---------------|
| 0 | $\frac{7}{3}$ |
| 1 | $\frac{7}{3}$ |
| 0 | $\frac{7}{3}$ |
| -1 | $\frac{5}{3}$ |
| 0 | $\frac{7}{3}$ |

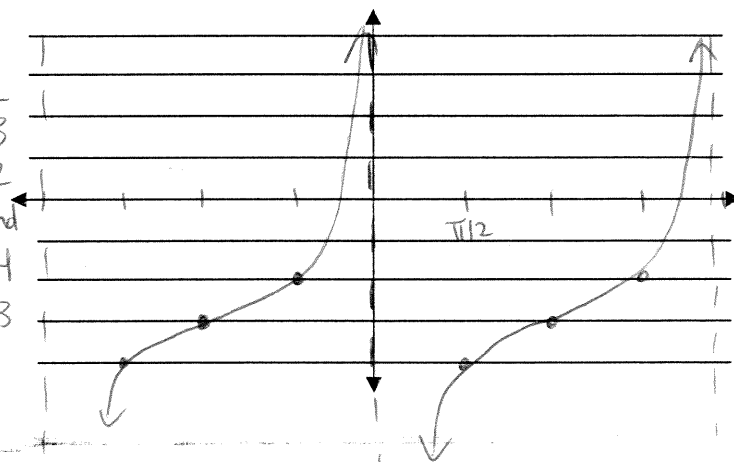


3. $y = \tan\left(\frac{1}{2}(x + \pi) - 3\right)$

| X | Y |
|----------|-----|
| 0 | 0 |
| $\pi/4$ | 1 |
| $\pi/2$ | und |
| $3\pi/4$ | -1 |
| π | 0 |

Amp. = none
Period = 2π
H. S. = $-\pi$
V. S. = -3

| y | Y |
|----------|-----|
| $-\pi$ | -3 |
| $-\pi/2$ | -2 |
| 0 | und |
| $\pi/2$ | -4 |
| π | -3 |



4. $y = 2 \csc\left(\frac{1}{3}x\right) - 6$

| | |
|----------|-----|
| 0 | und |
| $\pi/2$ | 1 |
| π | und |
| $3\pi/2$ | -1 |
| 2π | und |

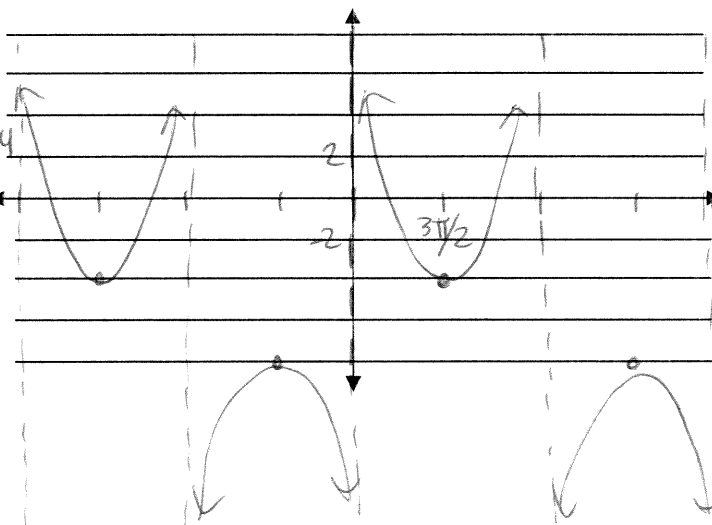
Amp. = none

Period = 6π

H. S. = 0

V. S. = -6

| | |
|----------|-----|
| 0 | und |
| $3\pi/2$ | -4 |
| 3π | und |
| $9\pi/2$ | -8 |
| 6π | und |



5. $y = 3 \sec\left(\frac{1}{2}\left(x - \frac{\pi}{4}\right)\right) + 2$

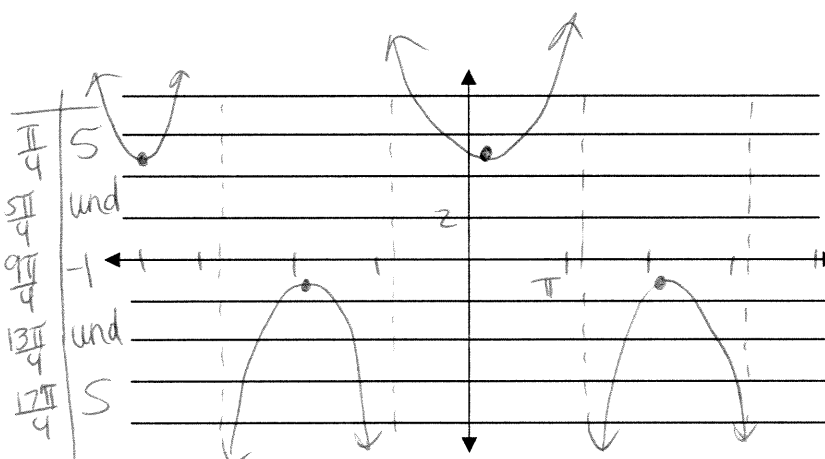
| | |
|----------|-----|
| 0 | 1 |
| $\pi/2$ | und |
| π | -1 |
| $3\pi/2$ | und |
| 2π | 1 |

Amp. = none

Period = 4π

H. S. = $\pi/4$

V. S. = 2



6. $y = \cot\left(\frac{1}{2}x\right) - 3$

| | |
|----------|-------|
| 0 | und |
| $\pi/4$ | 1 |
| $\pi/2$ | und 0 |
| $3\pi/4$ | -1 |
| π | und |

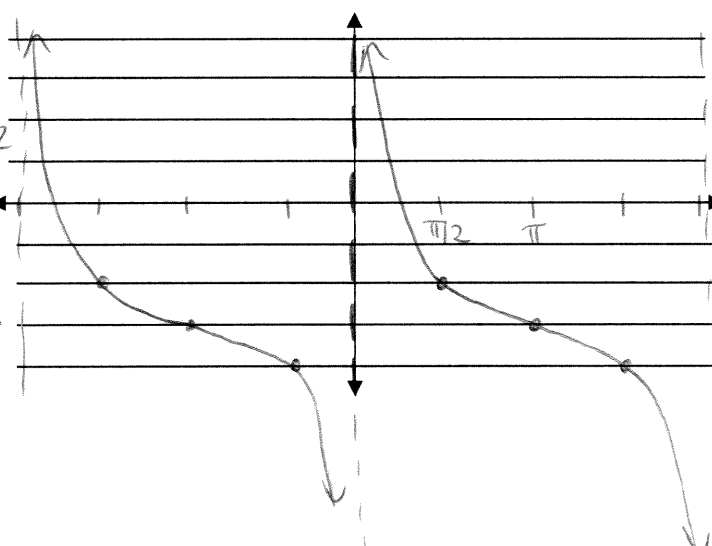
Amp. = none

Period = 2π

H. S. = none

V. S. = -3

| | |
|----------|-----|
| 0 | und |
| $\pi/2$ | -2 |
| π | -3 |
| $3\pi/2$ | -4 |
| 2π | und |



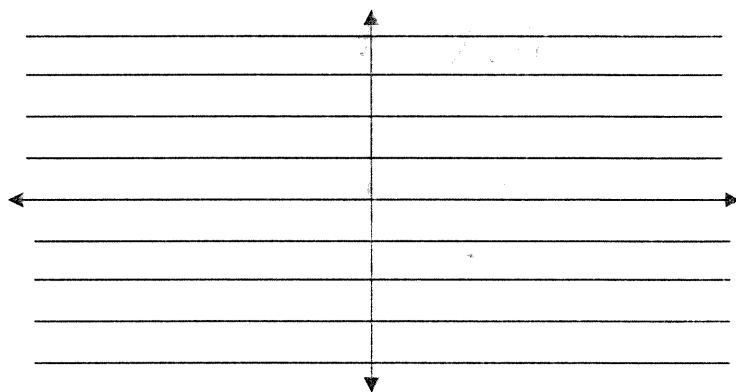
$$7. y = 3 \sin \frac{2\pi}{5}(x+1) + 3$$

Amp. = _____

Period = _____

H. S. = _____

V. S. = _____



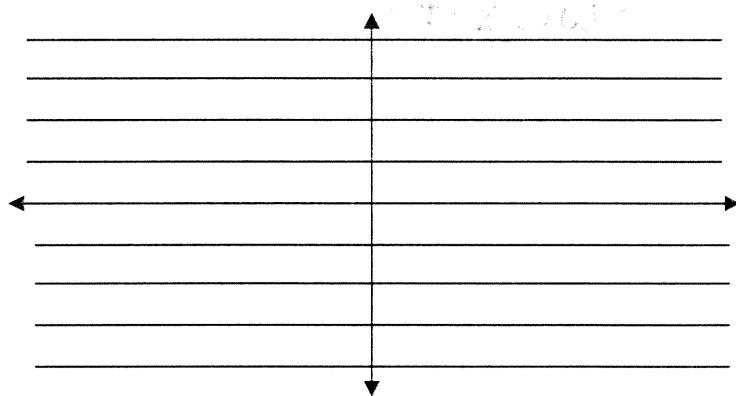
$$8. y = -2 \sec 2(x) + 3$$

Amp. = _____

Period = _____

H. S. = _____

V. S. = _____



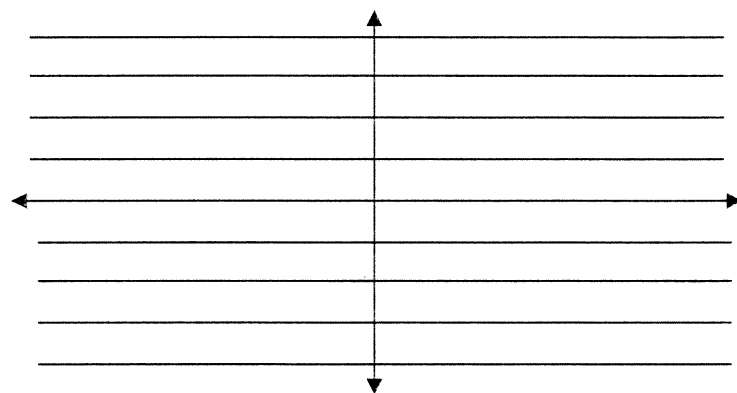
$$9. y = \tan \frac{1}{3}(x) - 3$$

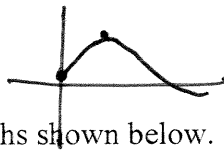
Amp. = _____

Period = _____

H. S. = _____

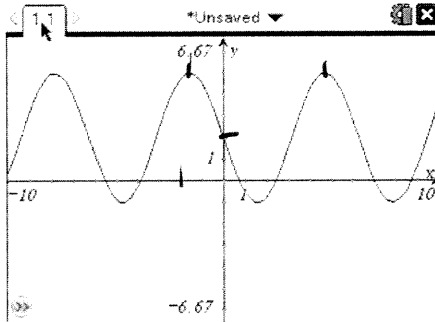
V. S. = _____



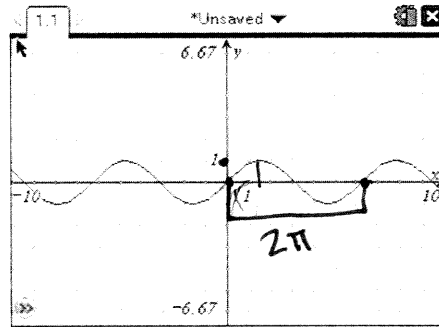


Write an equation for each of the graphs shown below. Use the indicated parent function.

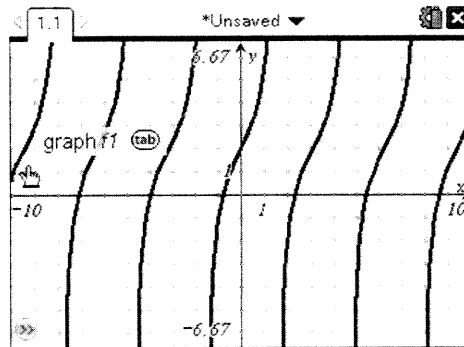
10. $3\sin x + 2$
 $y = \sin x$



11. $y = \cos(x - \frac{\pi}{2})$
 $y = \cos x$



12. $3\tan(x) + 2$
 $y = \tan x$



$$\frac{2\pi}{c} = \pi$$

13. $\sin \frac{3\pi}{4} =$ _____

14. $\tan \frac{3\pi}{2} =$ _____

15. $\cos \frac{7\pi}{6} =$ _____

16. $\sin \theta = -\frac{\sqrt{2}}{2}$, $\theta =$ _____ (θ in radians)

17. $\cos \theta = \frac{1}{2}$, $\theta =$ _____ (θ in radians)

18. $\sin \theta = -\frac{2}{5}$ and $\tan \theta > 0$, $\cos \theta =$ _____ and $\cot \theta =$ _____

19. $\tan \theta = -\frac{3}{8}$ and $\sin \theta > 0$, $\cos \theta =$ _____ and $\csc \theta =$ _____

20. $\cos \theta = -\frac{7}{10}$ and $\sin \theta > 0$, $\tan \theta =$ _____ and $\csc \theta =$ _____

Find the exact value of each expression.

1. $\sin(\sin^{-1} 0.567)$

0.567

2. $\cos^{-1}(\cos -\frac{3\pi}{2})$

 $\frac{\pi}{2}$

3. $\arctan(\tan 7\pi)$

0

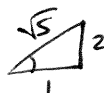
4. $\tan(\tan^{-1} 4)$

4

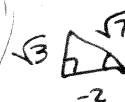
5. $\tan(\arcsin \frac{2}{3})$

 $\frac{2\sqrt{5}}{5}$ 

6. $\csc(\tan^{-1} 2)$

 $\frac{\sqrt{5}}{2}$ 

7. $\cot(\cos^{-1}(-\frac{\sqrt{3}}{2}))$

 $-\frac{2\sqrt{3}}{3}$ Find two solutions of the equation. Give your answers in degrees ($0^\circ \leq \beta < 360^\circ$) and in radians ($0 \leq \beta < 2\pi$).

8. $\cos \beta = \frac{\sqrt{3}}{2}$

 30°
 $\frac{\pi}{6}$ ~~330°~~
 ~~$\frac{11\pi}{6}$~~

9. $\csc \beta = -2$

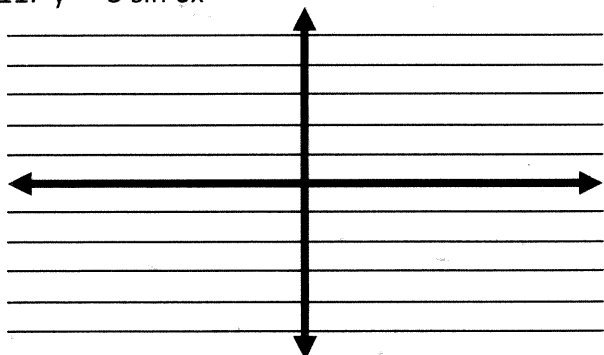
 $\frac{\pi}{3}$ 60° $\frac{5\pi}{3}$ ~~300°~~

10. $\cot \beta = -1$

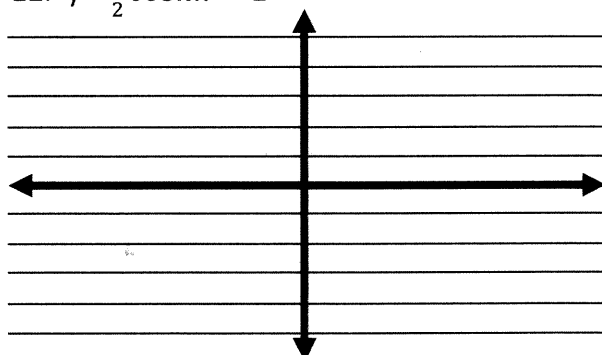
 $\frac{3\pi}{2}$ $\frac{3\pi}{4}$ 135° 270°

Graph each function. Label your axes and include a positive and a negative period.

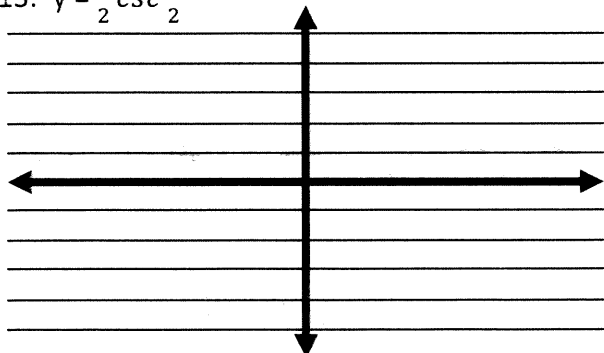
11. $y = -3 \sin 6x$



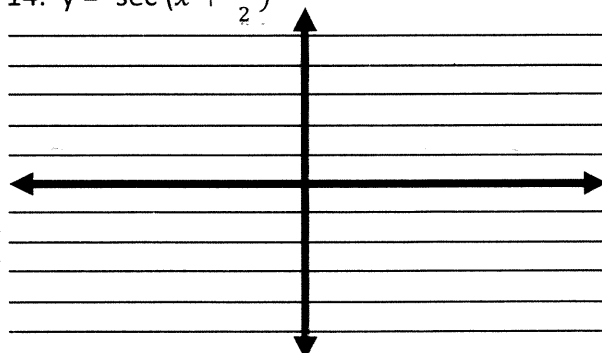
12. $y = \frac{1}{2} \cos \pi x - 2$



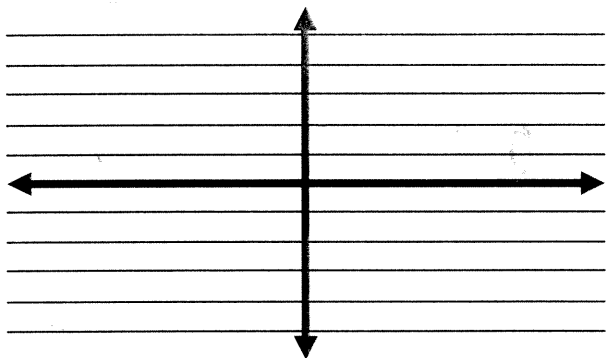
13. $y = \frac{3}{2} \csc \frac{x}{2}$



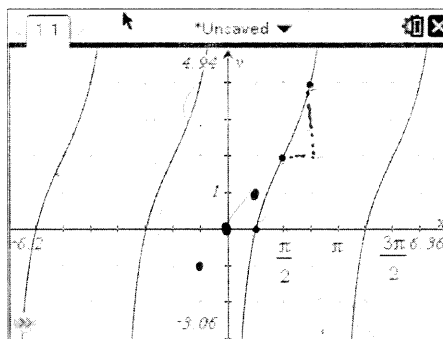
14. $y = -\sec(x + \frac{3\pi}{2})$



15. $y = \cot 2\pi x - 2$



16. Find an equation for the graph.



$\tan(x)$

$c=1$

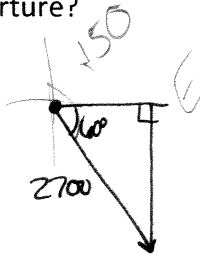
$a=2$

$h=\pi/2$

$k=+2$

$2 \tan(x - \frac{\pi}{2}) + 2$

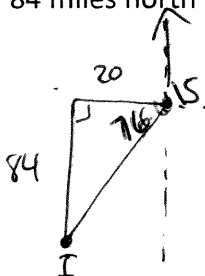
17. A plane is flying at 900 mph and has a bearing of 150° . After flying 3 hours, how far east and south will the plane have traveled from its point of departure?



east $\sin 60 = \frac{e}{2700} \approx 2338.27 \text{ ft}$

south $\cos 60 = \frac{s}{2700} \approx 1350 \text{ ft}$

18. A ship is 20 miles east and 84 miles north of an island. What bearing should be taken to sail directly to the island?



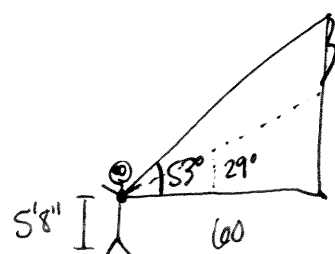
$\tan^{-1}(\frac{84}{20}) = 76.61$

Bearing: 193.39° or $S 13.39^\circ N$

19. Someone is at a parade looking up at a large balloon floating directly above the street. They are 60 feet from a point on the street directly beneath the balloon. The top of the balloon is at an angle of elevation of 53° . The bottom of the balloon is at an angle of elevation of 29° . At eye level the person is 5ft 8 in. How tall is the balloon?

$\tan 53^\circ = \frac{\text{large}}{60}$

$\tan 29^\circ = \frac{\text{small}}{60}$



large - small
 $60 \tan 53^\circ - 60 \tan 29^\circ$

46.36 Feet